Interpreting Shannon Entropy as a Bermudan Option Price

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The Shannon entropy of a discrete random variable is a well-known-measure of the width of the random variable's probability distribution. Likewise, the arbitrage-free prices of European and Bermudan options also measure the width of the distribution governing the price of its underlying security. In this paper, we interpret Shannon entropy as the arbitrage-free price of a particular kind of Bermudan option in a particular valuation model and under a natural centering and scaling transformation. The particular kind of Bermudan option is called a stoption. The particular valuation model is called the logistic model, because the discrete-time price process for the stoption's underlying security is a random walk with logistically distributed increments. In this logistic model, the stoption's price is given in closed form by the log sum exponential of a set of floors specified in the stoption's term sheet. The probabilities of exercising each day are also given in closed-form in this model. We center the stoption's floors by requiring ex ante that their probability weighted average vanish. We also scale these centered floors and the corresponding stoption's value by expressing both relative to the value of an option to exchange one logistically distributed random variable for another independent one. We then show that when the relative stoption's price is expressed in terms of the exercise probabilities, Shannon's formula for the entropy in nats emerges.

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